

Test: Rotational Mechanics**Multiple Choice: Choose the letter of the best answer. 3 points each.**

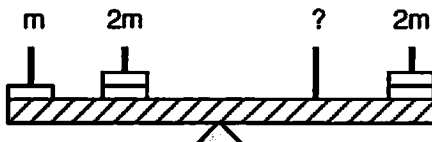
Questions 1 and 2 refer to the following:

The angular velocity as a function of time for an object is given by $\omega = 4t + 3$

1. C What is the angular acceleration of the object?
 a. 2 rad/s^2 . b. 3 rad/s^2 . c. 4 rad/s^2 . d. 8 rad/s^2 . e. 12 rad/s^2 .
2. D What is the angular displacement during the first 5 seconds of this motion?
 a. 15 rad. b. 20 rad. c. 50 rad. d. 65 rad. e. 130 rad.

$$\frac{3 + 23}{2} = \frac{26}{2} = 13$$

$$\frac{13}{65}$$



$$2m + 2m = x + 4m$$

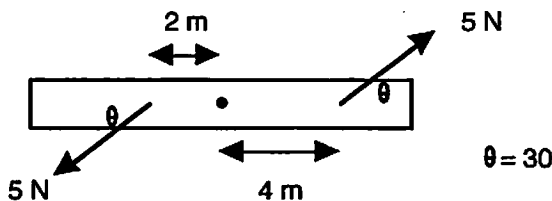
3. A In the figure above, what mass should be placed on the third peg so that the system will be balanced? (The outer pegs are twice as far from the balance point as the inner pegs.)
 a. 0 m. b. $1/2 m$. c. m. d. 2 m. e. none of these.

Problems 4 and 5 refer to the following:

A wheel constantly accelerates from rest, rotating 5 complete times in 60 seconds.

4. D What is the average angular velocity of the wheel?
 a. 0.083 rad/s . b. 0.17 rad/s . c. 0.26 rad/s . d. 0.52 rad/s . e. 1.05 rad/s .
5. A What is the angular acceleration (in rad/s^2) of the wheel?
 a. 0.0175. b. 0.167. c. 0.00278. d. 0.0055. e. 0.333.

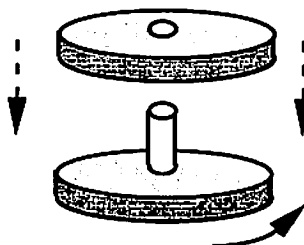
$$\rightarrow = 31.4 \text{ rad}$$



$$20 \sin 30 + 10 \sin 30$$

$$10 + 10 = 20$$

6. D A rod is pivoted about its center. A 5-N force is applied 4 m from the pivot and another 5-N force is applied 2 m from the pivot, as shown above. The magnitude of the total torque about the pivot (in $\text{N}\cdot\text{m}$) is:
 a. 0 b. 5 c. 8.7 d. 15 e. 26
7. C If a constant net torque is applied to an object, then the object will
 a. have a decreasing moment of inertia.
 b. rotate with constant angular velocity.
 c. rotate with constant angular acceleration.
 d. have an increasing moment of inertia.
8. A A wheel, mounted on a vertical shaft of negligible rotational inertia, is rotating at 500 rpm. Another identical, but not rotating, wheel is suddenly dropped onto the same shaft on top of the first wheel. The resultant combination of the two wheels will rotate at:
 a. 250 rpm. b. 354 rpm. c. 500 rpm.
 d. 707 rpm. e. 1000 rpm.



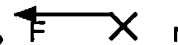
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9. A Imagine that you are sitting and rotating on a stool with your arms stretched out. What is conserved if you pull your arms in to your body?
- Angular momentum.
 - Rotational kinetic energy.
 - Angular velocity.
 - Only a and b are true.
 - Heck, all of the above are true.
10. A A disk with a rotational inertia of $5.0 \text{ kg}\cdot\text{m}^2$ and a radius of 0.25 m rotates on a frictionless fixed axis perpendicular to the disk and through its center. A force of 8.0 N is applied along the rotation axis. The angular acceleration of the disk is:
- 0
 - 0.40 rad/s^2
 - 0.60 rad/s^2
 - 1.0 rad/s^2
 - 2.5 rad/s^2
11. E Two rolling objects have the same mass and speed. Which has more kinetic energy?
- Don't be stupid, of course they have the same kinetic energy.
 - The one with the larger radius will have more kinetic energy.
 - The one with the smaller radius will have more kinetic energy.
 - The one with the smaller moment of inertia will have more kinetic energy.
 - The one with the larger moment of inertia will have more kinetic energy.

Problems 12 to 13 refer to the following possible choices and are only 1 point each:

- a. \rightarrow b. \leftarrow c. \uparrow d. \downarrow e. \bullet f. \times g. 0

12. C What is the direction of the torque in the picture to the right?



13. F What is the direction of the torque in the picture to the right?



$11 @ 3 = 33$
 $+ 2 = 35$
 $5 @ 12 = 60$
 95

Test: Rotational Mechanics**Problem Solving: Show all work.** 12 @

14. A solid sphere is rolling without slipping with a speed of 6 m/s when it encounters a hill of base angle 30°. How high up the hill does the sphere roll?



$$h = \frac{7v^2}{10g} = \frac{7(6)^2}{10 \cdot 10}$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = mgh$$

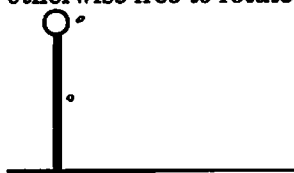
$$\frac{1}{2}mv^2 + \frac{1}{2}\left[\frac{2}{5}mr^2\right]\omega^2 = mgh$$

$$h = 2.52 \text{ m}$$

$$\frac{1}{2}v^2 + \frac{1}{5}v^2 = gh$$

$$\frac{7}{10}v^2 = gh$$

15. A thin rod with a length of 0.8 meters is standing perfectly upright. There is a small mass attached to the very end of the rod. The rod and the small mass have the same mass. If it were to be knocked over (with a tiny nudge) what would be the linear speed of the mass at the end of the rod just as it hits the ground? Assume the other end of the rod is fixed in place, but otherwise free to rotate.



$$mgL + mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2$$

$$\frac{3}{2}mgL = \frac{1}{2}\left[ML^2 + \frac{1}{3}ML^2\right]\omega^2$$

$$3g = \left[L + \frac{1}{3}L\right]\omega^2$$

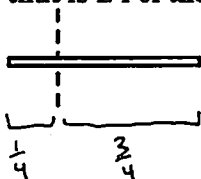
$$3g = \frac{4}{3}L\omega^2$$

$$\omega^2 = \frac{9}{4} \frac{g}{L}$$

$$V = 4.24 \text{ m/s}$$

$$V = L\omega = L\sqrt{\frac{9}{4} \frac{g}{L}} = \frac{3}{2}\sqrt{gL} = \frac{3}{2}\sqrt{10 \cdot 0.8}$$

16. What is the moment of inertia of a 20 kg thin rod of length 3 meters if it is rotated about an axis that is 1/4 of the way from its end?



$$\int_{-1/4}^{3/4} \frac{M}{L} x^2 dx = \frac{1}{3} \frac{M}{L} x^3 \Big|_{-1/4}^{3/4} = \frac{M}{3L} \left[\left(\frac{3}{4}\right)^3 L^3 - \left(-\frac{1}{4}\right)^3 L^3 \right]$$

$$= \frac{M}{3L} \left[\frac{27}{64} L^3 + \frac{1}{64} L^3 \right]$$

$$= \frac{1}{3} ML^2 \left[\frac{27}{64} + \frac{1}{64} \right]$$

$$= \frac{7}{3 \cdot 16} ML^2$$

$$= \frac{7}{48} ML^2$$

side 3

$$\begin{aligned} \text{or } \frac{1}{3} \left(\frac{1}{4} M \right) \left(\frac{1}{4} L \right)^2 + \frac{1}{3} \left(\frac{3}{4} M \right) \left(\frac{3}{4} L \right)^2 \\ = \frac{1}{192} ML^2 + \frac{27}{192} ML^2 = \frac{28}{192} ML^2 \\ = \frac{7}{48} ML^2 \end{aligned}$$

$$= \frac{7}{48} (20)(3)^2 = 26.25 \text{ kg} \cdot \text{m}^2$$

$$\frac{28}{64} = \frac{7}{16}$$

b

Test: Rotational Mechanics

17. A 0.2 kg bug is at the center of a wheel 0.4 m radius 0.6 kg wheel that is rotating at 25 rpm. The bug carefully crawls out to the edge of the wheel. What is the linear speed of the bug when it gets to the edge of the wheel. (Consider the wheel a hoop.)

$$L_i = L_f$$

$$[MR^2 + 0] \omega_i = [MR^2 + mR^2] \omega_f$$

$$[(.6)(.4)^2] 25 = [(.6)(.4)^2 + (.2)(.4)^2] \omega_f$$

$$(.096)(25) = [.096 + .032] \omega_f$$

$$[.096](25) = [.096 + .032] \omega_f$$

$$(.096)(25) = (.13) \omega_f$$

$$\omega_f = 18.75 \text{ rad/s rpm}$$

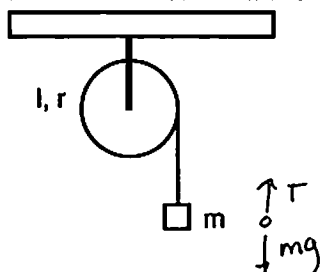
$$\left(\frac{25 \text{ rev}}{\text{min}} \right) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 2.62 \text{ rad/s}$$

$$(18.75) \frac{\text{rev}}{\text{min}} \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{1 \text{ min}}{60 \text{ s}} \right) = 1.96 \text{ rad/s}$$

$$V = R\omega = (.4)(1.96) = .79 \text{ m/s}$$

$$\boxed{V = .79 \text{ m/s}}$$

18. A 3-kg block is attached to a cord that is wrapped around the rim of a flywheel of radius 0.40 m and hangs vertically, as shown. The rotational inertia of the flywheel is 0.50 kg·m². When the block is released and the cord unwinds, what is the acceleration of the block?



$$\sum \tau = I\alpha$$

$$\sum F = ma$$

$$mg - T = ma$$

$$\boxed{T = mg - ma}$$

$$rmg - rma = \frac{Ia}{r}$$

$$r^2mg - r^2ma = Ia$$

$$r^2mg = Ia + r^2ma$$

$$a = \frac{r^2mg}{I + mr^2}$$

$$rT = I\alpha$$

$$r(mg - ma) = Ia$$

$$r(mg - ma) = I \frac{a}{r}$$

$$Ia = mr^2g - mr^2a$$

$$a = \frac{mr^2g}{I + mr^2}$$

$$a = \frac{(3)(.4)^2(10)}{.5 + (3)(.4)^2} = \frac{4.8}{.98}$$

$$\boxed{a = 4.9 \text{ m/s}^2}$$